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Complexity Reduction Techniques for MMSE Channel Estimator in OFDM

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Abstract: Channel Estimation in Orthogonal Frequency Division Multiplexing (OFDM) is difficult task in any wireless technology. In pilot based channel estimation, choice of estimators is important which involves trade-offs between complexity and accurate estimation. The Least Square (LS) has degrading performance due to AWGN noise, although it exhibits lower complexity. The LS estimation performance can be improved using Minimum Mean Square Error (MMSE) estimator however it executes large number of operations with increasing number of sub-carriers. In this paper, simplified low complexity MMSE estimators (SMMSE) are presented. Low complexity SMMSE estimators based on various decomposition techniques like Singular Value Decomposition (SVD) and QR Decomposition (QRD) are proposed. The complexity and Bit Error Rate performance of all estimators are presented to evaluate efficiency of estimators.

Keywords: OFDM, LS, MMSE, SMMSE, SVD, QRD.

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I.INTRODUCTION

OFDM transceiver system is widely applied for modern wireless mobile communications such as 4G, 5G etc. The heart of OFDM receiver is the channel estimation technique. Efficient channel estimation has a direct impact on the bit error rate (BER) performance of the OFDM system. There are two main problems in designing channel estimators for wireless mobile OFDM systems. The first problem is the arrangement of pilot information, where pilot means the reference signal used by both transmitters and receivers and the second problem is the design of an estimator with both low complexity and good performance. The most well-known and widely used training-based channel estimations are; LS (Least square), MMSE (Minimum Mean Square Error), LMS (least mean square), RLS (Recursive Least Square). Paper [2] describes details of LS and MMSE estimators. Results show better performance of MMSE over LS. However MMSE is highly complex. In paper [3] block pilot assisted LS, MMSE, OLR-MMSE estimators with and without decision feedback equalizer are described. Authors presented comb type pilot assisted maximum likelihood and PCMB estimator. In block type method, OLR MMSE with decision feedback equalizer gives better performance among all and in comb type estimator PCMB yields best performance. Authors proposed various denoising methods in paper [4] to reduce complexity of LS and MMSE estimators. In paper [5], authors investigated low complexity time domain linear MMSE estimator for space-frequency OFDM (SF-OFDM) and space-time OFDM (ST-OFDM) systems. They employed a convenient representation of the channel impulse responses based on the Karhunen-Loeve (KL) orthogonal expansion and found MMSE estimates of the uncorrelated KL series expansion coefficients. QR decomposition is an alternative method to reduce the complexity of inverse matrices. Paper [6] describes how the complexity of LS is reduced using QR decomposition. To improve performance of LS and to reduce complexity of MMSE, simplified LS (SLS) and SMMSE estimators are proposed in paper [7]. The SLS outperforms the conventional LS method for a range of SNRs with relatively higher complexity than LS. SMMSE has less complexity while giving degrading performance than MMSE. In paper [8], authors have compared two solutions for the complexity reduction of LMMSE in OFDM/OQAM. The results revealed that both proposed approximations have very similar performances, but Approximation-1 is more complex than Approximation-2. In [9], concept of "virtual pilot" is proposed to reduce the complexity of the conventional MMSE channel estimator while at the same time maintaining the high performance under critical time and frequency selective channels. MMSE for pilots and virtual pilots are performed within small sub blocks to achieve low complexity. Low complexity partial-sampled MMSE channel estimation is presented in paper [10] for compromising between complexity and performance. Authors reduced MMSE channel estimation complexity by partially sampling the MMSE weight matrix. The simulation results showed that the bit error rate (BER) performance and equalized signal constellation scatter plot significantly improved over the least square channel estimation and had comparable BER performance with MMSE channel estimation. In this study, reduced complexity approximations of MMSE (SMMSE) estimators are proposed. The complexity of SMMSE is reduced using SVD decomposition method. Further, QR decomposition is used to reduce complexity of SMMSE. The performance of all simplified MMSE estimators are compared based on Bit Error Rate.

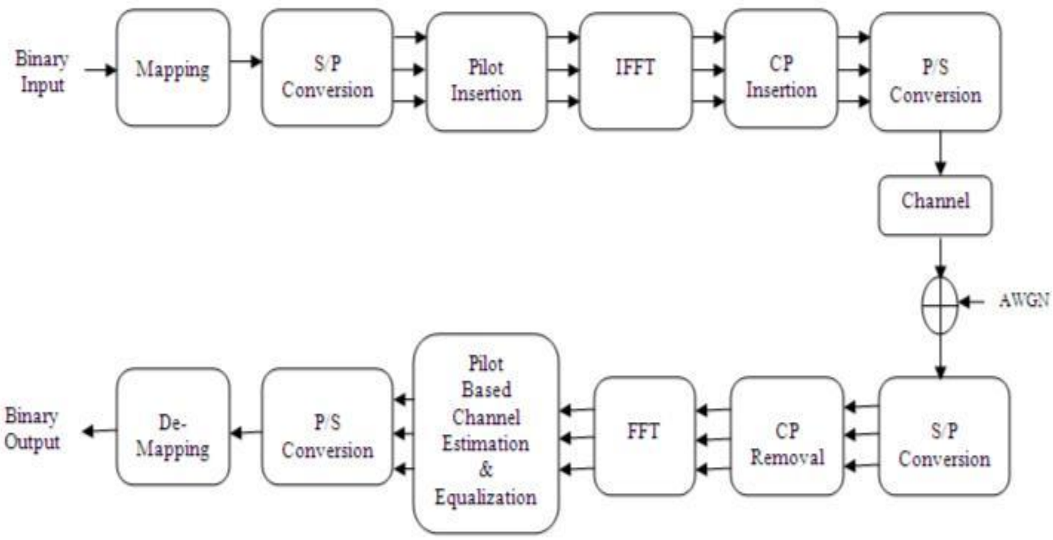

Fig.1. Block Diagram of OFDM System

II. SYSTEM DESCRIPTION

The baseband model for a typical OFDM wireless communication system is shown in Fig. 1. The binary data to be transmitted is first mapped using digital modulation scheme 16-quadrature amplitude modulation (QAM). The 16-QAM modulated data symbols are converted from serial to parallel form. One column of data symbols is a single OFDM symbol. Each row of data symbols is termed as subcarrier. If one OFDM symbol comprises of N data symbols, the number of subcarriers are N. The comb-pilot insertion is used to transmit known data, which assist in estimating the channel at the receiver. Each OFDM symbol is converted to time domain using an inverse IFFT. A cyclic prefix (CP) is attached at the beginning of each time domain OFDM symbol to remove ISI in the channel. After the insertion of Cyclic Prefix, the resultant OFDM symbols are converted into serial form and are transmitted through a multipath channel. The channel selected is Rayleigh multipath channel. An additive white Gaussian noise (AWGN) is added. At the receiver, the received symbols are converted to parallel form and the CP is removed. The resulting OFDM symbols are sent through a FFT block and the corresponding frequency domain symbols are obtained. A pilot based equalizer uses these symbols to estimate the channel and further processes the received data to estimate transmitted OFDM symbols. There symbols are then converted to serial form and are de-mapped, to obtain the transmitted binary data.

In baseband model of a pilot-based OFDM we use the following variables: *y* the received signal vector, *x* the transmitted signal vector, *h* the Channel Impulse Response (CIR), and *v* the AWGN vector. An *N*-point IFFT is used to modulate the OFDM signal.
The channel is modeled as an impulse response *h (t):*

$$h(t) = \sum_{m=0}^{N-1} \alpha_m \, \delta(\tau - \tau_m T_s) \qquad (1)$$

The received in vector form is given by,

$$y = x \circledast h + v \qquad (2)$$

The FFT of received signal y in vector form is:

$$Y = XH + V \qquad (3)$$

III. CHANNEL ESTIMATION

The estimations of channel coefficients based on the comb-type pilot arrangement with least square (LS), minimum mean-square error (MMSE) are presented here. To recover data information symbols, spline interpolation is used.

A. Least Square (LS) Channel Estimation

The channel impulse responses (CIR) by Least Square estimator is given by,

$$\hat{H}_{LS} = Y/X \; = Y_p/X_p \qquad (4)$$

Where, Y_p and X_p are outputs and inputs at pilot locations.

The advantage of LS algorithm is its simplicity and without using any knowledge of the statistics of the channels, the LS estimators are calculated with very low complexity.

B. Minimum Mean Square Error (MMSE) Channel Estimation

The MMSE estimator employs the second-order statistics of the channel conditions to minimize the MSE. The channel impulse response for MMSE estimator can be calculated as,

$$\begin{aligned}\hat{H}_{MMSE} &= R_{HY}R_{YY}{}^{-1}Y \\ &= R_{HH}X^H(XR_{HH}X^H + \sigma_N^2 I_N)^{-1}X\hat{H}_{LS} \\ &= R_{HH}(R_{HH} + \sigma_N^2(X^H X)^{-1})^{-1}\hat{H}_{LS}\end{aligned} \quad (5)$$

Reduced complexity approximations of MMSE estimators are explained in detail in the following sections.

C. Simplified MMSE (SMMSE)

In equation (5) $(X^H X)^{-1}$ this term should be calculated at each frame. Calculation time of equation (5) can be reduced by replacing $(X^H X)^{-1}$ with its expectation $E\{(X^H X)^{-1}\}$. It's assumed that the same signal constellations are on all tones and have equal probability on all constellation points. Thus we have,

$E\{(X^H X)^{-1}\}=E\{|1/X_k|^2\}I.$ (6)

The average SNR is defined, $SNR = E\{|1/X_k|^2\}/\sigma_N^2$, and the term $\beta = E\{|X_k|^2\}/E\{|1/X_k|^2\}$. The term $\sigma_N^2(X^H X)^{-1}$ is then approximate by$(\beta/SNR)\, I$, where β is a constant depending only on the signal constellation. For example, for a 16-QAM, $\beta = 17/9$.

Thus, we obtain simplified MMSE estimator as,

$$\hat{H}_{SMMSE} = R_{HH}(R_{HH} + \frac{\beta}{\mathrm{SNR}}\, I)^{-1}\hat{H}_{LS} \quad (7)$$

Equation (11) can be rewritten as,

$$\hat{H}_{SMMSE} = W_{simplified}\hat{H}_{LS} \quad (8)$$

Where, the coefficient $W_{simplified} = R_{HH}(R_{HH} + \frac{\beta}{\mathrm{SNR}}\, I)^{-1}$. The matrix$(R_{HH} + \frac{\beta}{\mathrm{SNR}}\, I)^{-1}$ needs to be calculated only once. Under these conditions the estimation requires n=*N* multiplications per block. To further reduce the complexity of the estimator, we use low-rank R_{HH} matrix in equation (7). In equation (1), most of the energy in *h*, lies in the first *L* no. of samples, where *L* is a small fraction of *h*.

To find such energy contained samples, *L* is calculated by, $L = 1 + (N * T_G/T_S)$, where T_G and *Ts* represent cyclic extension and sampling interval of the OFDM system, respectively. The ideal value of T_G/T_S is often chosen, as recommended by IEEE 802.11 and IEEE 802.16, among $\{1/32, 1/16, 1/8\}$.For example, if the FFT size is 64 and T_G/T_S is set to $1/8$, then *L* is found to be as small as 9. This implies, the significant portion of the channel energy is contained in first eight samples, while the remaining samples are mostly filled with unwanted noise. Thus, the proposed SMMSE excludes all the low energy taps for channel estimation and reduces the size of $W_{simplified}$' to $(L \times L)$. In this study, three simplified MMSE estimators SMMSE-17, SMMSE-25, SMMSE-35and full length SMMSE (i.e. with total number of subcarriers N)

D. Singular Value Decomposition of SMMSE (SVD-SMMSE)

To further reduce the complexity of SMMSE, SVD algorithm is considered. In this algorithm, SVD of channel autocorrelation matrix (R_{HH}) is taken into account. The SVD of R_{HH} is given as:

$$R_{HH} = U\Lambda U^H$$

Where, U is a unitary matrix containing singular vectors and Λ is a diagonal matrix containing the singular values of R_{HH} given by $\lambda_0 \geq \lambda_1 \geq \ldots\ldots\lambda_{N-1}$ on its diagonal.

Thus, equation (7) of SMMSE can be written as,

$$\hat{H}_{SMMSE-SVD} = U\Lambda U^H(U\Lambda U^H + \frac{\beta}{\mathrm{SNR}}\, I)^{-1}\hat{H}_{LS} \quad (9)$$

Let us consider, $\Delta = \Lambda(U\Lambda U^H + \frac{\beta}{\mathrm{SNR}}\, I)^{-1}$

This can be written as, $\Delta = \frac{\lambda_k}{\lambda_k + \frac{\beta}{SNR}}$

In low rank SMMSE-SVD only diagonal values are considered. So, Δ could be written with entries,

$$\delta_k = \frac{\lambda_k}{\lambda_k + \frac{\beta}{SNR}} \qquad k = 0,1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots L-1.$$

$$= 0 \qquad k = L, L+1 \ldots\ldots\ldots\ldots\ldots N-1.$$

Where, L is optimal low rank of R_{HH}. It is again decided using the formula, $L = 1 + (N * T_G / T_S)$.

Finally, OLR-SVD is represented as,

$$\hat{H}_{SMMSE-SVD} = U \Delta U^H \hat{H}_{LS}$$

$$= U \begin{bmatrix} \Delta_L & 0 \\ 0 & 0 \end{bmatrix} U^H \hat{H}_{LS} \qquad (10)$$

Where, Δ_L is $L \times L$ diagonal matrix.

$$\hat{H}_{SMMSE-SVD} = W_{SVD} \hat{H}_{LS} \qquad (11)$$

Where, $W_{SVD} = U \begin{bmatrix} \Delta_L & 0 \\ 0 & 0 \end{bmatrix} U^H$

In this paper, three optimal low rank SVD SMMME estimators are considered viz. SMMSE-SVD-17, SMMSE-SVD-25, SMMSE-SVD-35.

E. QR Decomposition of SMMSE (QRD-SMMSE)

In this study, to reduce complexity of SMMSE another decomposition method, QR decomposition is applied on channel autocorrelation matrix (R_{HH}). The QR decomposition of R_{HH} is given by, $R_{HH} = Q.R$. Where, Q is unitary matrix. Hence, $Q^{-1} = Q^H$ and $Q.Q^H = I$. Here, R is upper triangular matrix.

Putting $R_{HH} = Q.R$ in equation (7) we have,

$$\hat{H}_{SMMSE-QRD} = Q.R(Q.R + \frac{\beta}{SNR} I)^{-1} \hat{H}_{LS} \qquad (12)$$

Multiplying by Q on both sides,

$$Q.\hat{H}_{SMMSE-QRD} = Q.Q.R(Q.R + \frac{\beta}{SNR} I)^{-1} \hat{H}_{LS}$$

On the right side, Q out of bracket becomes Q^H in the bracket. Hence we get,

$$Q.\hat{H}_{SMMSE-QRD} = Q.R \left(Q^H.Q.R + Q^H.\frac{\beta}{SNR} I \right)^{-1} \hat{H}_{LS}$$

Rearranging above equation we have,

$$\hat{H}_{SMMSE-QRD} = Q^H.Q.R(Q^H.Q.R + \frac{\beta}{SNR} Q^H.I)^{-1} \hat{H}_{LS} \qquad (13)$$

However, $Q.Q^H = I$. Thus, equation (13) becomes,

$$\hat{H}_{SMMSE-QRD} = I.R(I.R + \frac{\beta}{\text{SNR}} Q^H.I)^{-1}\hat{H}_{LS} \tag{14}$$

$$\hat{H}_{SMMSE-QRD} = W_{QRD}\,\hat{H}_{LS} \tag{15}$$

Where, the coefficient $W_{QRD} = I.R(I.R + \frac{\beta}{\text{SNR}} Q^H.I)^{-1}$. Due to multiplication of identity matrix in each term of equation (13), the complexity becomes lower than that of equation (7). Here, we can have again three optimal low rank matrices of QRD SMMSE estimators, viz. SMMSE-QRD-17, SMMSE-QRD-25, and SMMSE-QRD-35.

IV. COMUTATIONAL COMPLEXITY OF ESTIAMTORS

Table.1. below shows the computational complexity of all variables. The complexity is described in terms of number of operations and it is denoted by symbol O. In this study, n represents the total subcarriers in one OFDM symbol and complexity is calculated for one OFDM symbol in a block. As number of symbols increases in a block, $(X^H X)^{-1}$ is to be calculated for each symbol and complexity increases, which is calculated only once in case of remaining algorithms.

Table.1. Computational Complexity of various variables

Variables	**Complexity** (In terms of Number of Operations)
$O(R_{HH})$	n^2
$O(\sigma_N^2(X^H X)^{-1})$	n^2
$O(\frac{\beta}{\text{SNR}} I)$	n^2
$O(U\Delta U^H)$	$2n^3$
$O(I.R)$	n^2
$O(\hat{H}_{LS})$	$n(1+n)$

Table.2. Computational Complexity of Estimators

Estimators	**Total Complexity** (In terms of total Number of operations)
LS	$n(1+n)$
MMSE	$2n^3+2n^2+3n$
SMMSE	$2n^3+2n^2+2n$
SVD-SMMSE	$2n^3+n^2+n$
QRD-SMMSE	n^3+3n^2+3n

Table.2. describes the complexity of estimators for only one symbol in a block. As the rank of matrices decreases, the complexity of estimators also decreases. However, it degrades the performance as some information sub carriers are not to be considered.

V. SIMULATION AND RESULTS

In the simulation, 64 numbers of subcarriers, 64-point FFT, 16-QAM modulation, 4 numbers of multi-paths, Rayleigh channel model, comb type pilot insertion, spline interpolation, 16 numbers of cyclic prefix are used. The performance of each estimator is presented in terms of Bit Error rate (BER). Fig.1. shows the performance of LS and simplified MMSE estimators. It is observed that SMMSE estimator with full rank has better performance than LS. As rank decreases the performance of SMMSE is improved. In Fig.3.BER of SVD-SMMSE with different ranks is presented.

As rank decreases the performance decreases. Full rank SVD-SMMSE-64 estimator has slightly better performance than that of SMMSE. However, the computational complexity is lower. Fig.4. shows the performance of QRD-SMMSE estimators for different ranks. As rank decreases, the performance of estimator decreases. Performance of QRD-SMMSE estimator for full rank is slightly better than that of SMMSE. However, the computational complexity is lower. Fig.5. shows performance of SVD-SMMSE and QRD-SMMSE for full rank. It is observed that performance of both estimators is same but slightly improved than that of SMMSE estimator.

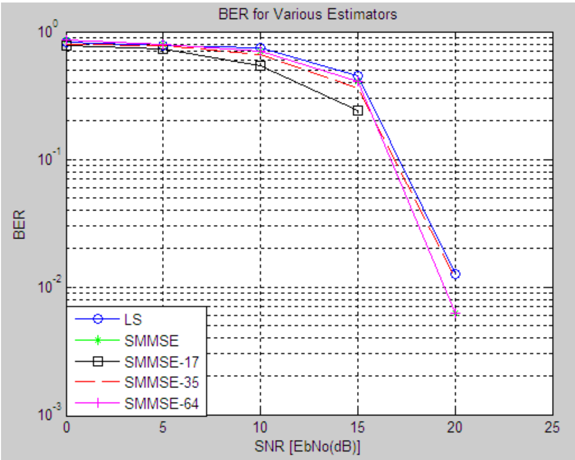

Fig.2. BER vs. SNR of LS and Simplified MMSE estimators

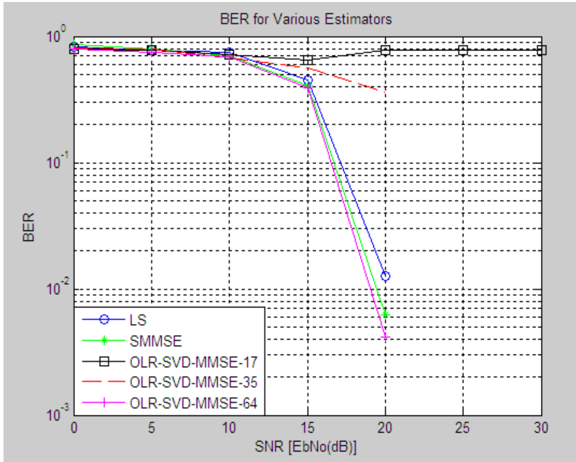

Fig.3. BER vs.SNR of SVD-SMMSE estimators

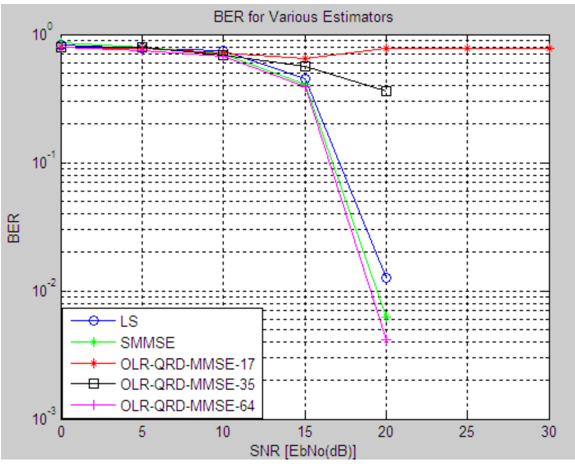

Fig.4. BER vs.SNR of QRD-SMMSE estimators

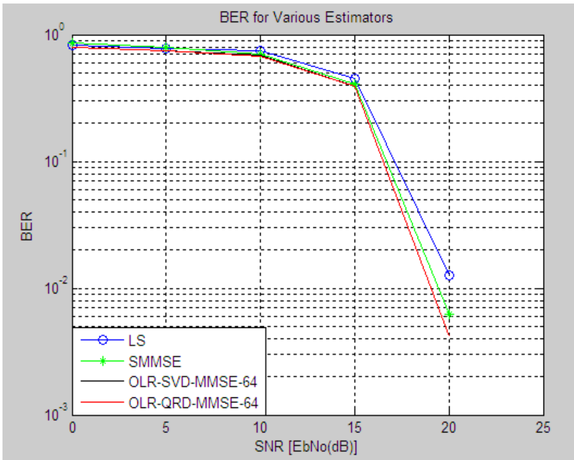

Fig.5. BER vs. SNR of SVD-SMMSE and QRD-SMMSE

VI. CONCLUSION

This paper compares LS, MMSE and different low complexity simplified versions of MMSE estimators. SMMSE estimator has same performance as that of MMSE with low complexity. SVD-SMMSE and QRD-SMMSE estimators slightly improve performances as compared to SMMSE. As rank of respective matrices decreases, the performance of SVD-SMMSE and QRD-SMMSE estimators degrades. The complexity of SMMSE is less than that of MMSE. The complexity of SVD-SMMSE is again less than SMMSE and that of QRD-SMMSE is less than that of SVD-SMMSE estimator. Thus, in terms of complexity and performance, QRD-SMMSE is better for large number of sub-carriers as well as OFDM symbols per block.

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